Measuring r^*: A Note on Transitory Shocks

Kurt F. Lewis Francisco Vazquez-Grande

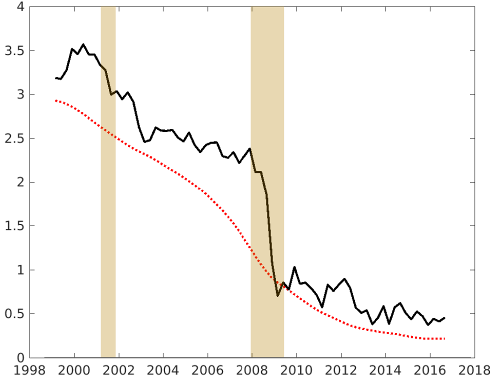
Federal Reserve Board

Federal Reserve Disclaimer

The analysis and conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors.

Chair Yellen, March 15, 2017

"That's based on our view that the neutral nominal federal funds rate...is currently quite low by historical standards. That means that the federal funds rate does not have to rise all that much to get to a neutral policy stance."



Main Results

We incorporate fully the uncertainty on r^* embedded in the data, we find:

- a more prociclycal r^* in the benchmark model.
- evidence of transitory shocks to r^*.

Baseline model (Holston, Laubach and Williams 2016)

$$
\begin{aligned}
\widetilde{y}_t &= y_t - y_t^* \\
\widetilde{y}_t &= a_{y,1}\widetilde{y}_{t-1} + a_{y,2}\widetilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}\left(r_{t-j} - r_{t-j}^*\right) + \varepsilon_{\widetilde{y},t} \\
\pi_t &= b_\pi \pi_{t-1} + (1 - b_\pi)\pi_{t-2,4} + b_Y \widetilde{y}_{t-1} + \varepsilon_{\pi,t} \\
y_t^* &= y_{t-1}^* + g_{t-1} + \varepsilon_{y^*,t} \\
g_t &= g_{t-1} + \varepsilon_{g,t} \\
r_t^* &= g_t + z_t \\
z_t &= z_{t-1} + \varepsilon_{z,t}
\end{aligned}
$$

Extended model

$$
\begin{aligned}
\widetilde{y}_t &= y_t - y_t^* \\
\widetilde{y}_t &= a_{y,1}\widetilde{y}_{t-1} + a_{y,2}\widetilde{y}_{t-2} + \frac{a_r}{2}\sum_{j=1}^{2}\left(r_{t-j} - r_{t-j}^*\right) + \varepsilon_{\widetilde{y},t} \\
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y_t^* &= y_{t-1}^* + g_{t-1} + \varepsilon_{y^*,t} \\
g_t &= \mu_g + \rho_z(g_{t-1} - \mu_g) + \varepsilon_{g,t} \\
r_t^* &= g_t + z_t \\
z_t &= \rho_z z_{t-1} + \varepsilon_{z,t}
\end{aligned}
$$

Model estimation: MLE

- pileup problem (Stock 1994)
- Solution proposed by Laubach and Williams 2003
  based on medium unbiased estimator from Stock and Watson 1998:

$$\lambda_g \equiv \frac{\sigma_g}{\sigma_{y^*}} , \quad \lambda_z \equiv \frac{a_r \sigma_z}{\sigma_{\tilde{y}}}.$$

LW Method:

Step 1 Simplify model, estimate λ_g.

Step 2 Fix λ_g value, use alternative simplification, estimate λ_z.

Step 3 Fix λ_g and λ_z, estimate remaining parameters.

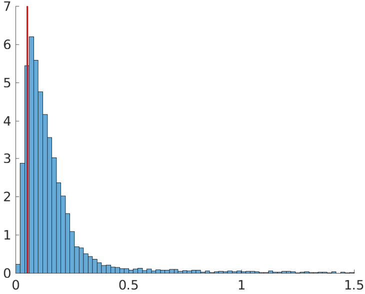
Model estimation: MCMC

- pileup problem? (DeJong and Whiteman 1993, Kim and Kim 2017)
- We use standard Bayesian methods (random walk MC, FFBS).
- Flat priors.
- Imposing HLW λ_g and λ_z, we replicate HLW.

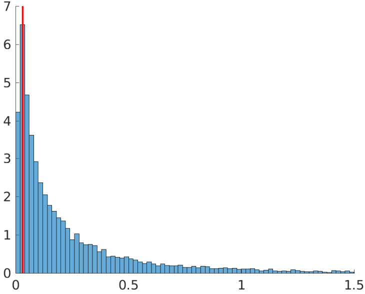
Priors

Name	Domain	Density	Parameter 1	Parameter 2
a_1	$\mathbb{R}$	Normal	0	2
a_2	$\mathbb{R}$	Normal	0	2
a_r	$\mathbb{R}^-$	Normal	0	2
b_1	$[0, 1]$	Uniform	0	1
b_Y	$\mathbb{R}^+$	Normal	0	2
ρ_g	$\mathbb{R}$	Normal	1	$\frac{1}{2}$
μ_g	$\mathbb{R}$	Normal	0	2
ρ_z	$\mathbb{R}$	Normal	1	$\frac{1}{2}$
σ_1	$[0, 5]$	Uniform	0	5
σ_2	$[0, 5]$	Uniform	0	5
σ_3	$[0, 5]$	Uniform	0	5
σ_4	$[0, 5]$	Uniform	0	5
σ_5	$[0, 5]$	Uniform	0	5

MLE vs Bayesian: the difference in $\lambda's$



λ_g



λ_z

Models Considered

- We estimate the key parameters ρ_g, μ_g and ρ_z of the extended model. We consider four alternatives:

Model I	$\rho_g = 1$, $\rho_z = 1$, (only different estimation technique)
Model II	ρ_g and μ_g estimated, $\rho_z = 1$
Model III	$\rho_g = 1$, ρ_z estimated
Model IV	ρ_g, μ_g and ρ_z estimated

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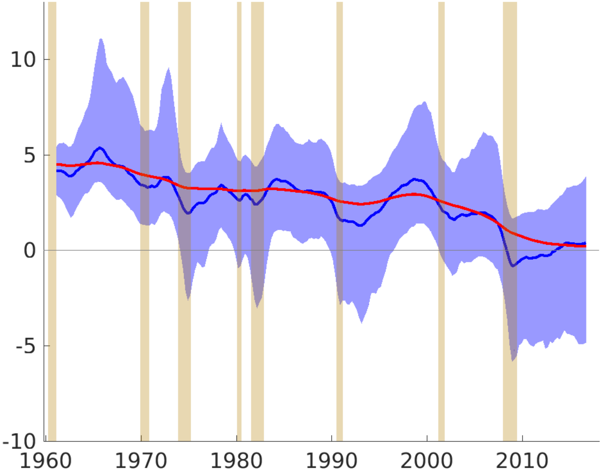
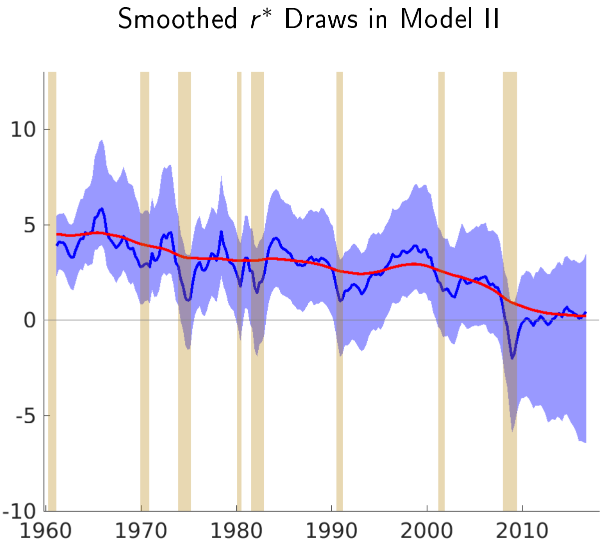
Model I: $\rho_g = 1$, $\rho_z = 1$

Smoothed r^* Draws in Model I (Shaded 10th to 90th Percentile)

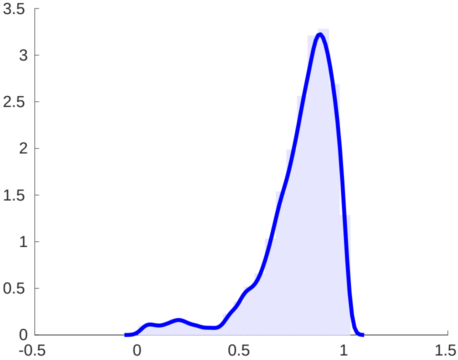


Model II: ρ_g and μ_g estimated, $\rho_z = 1$

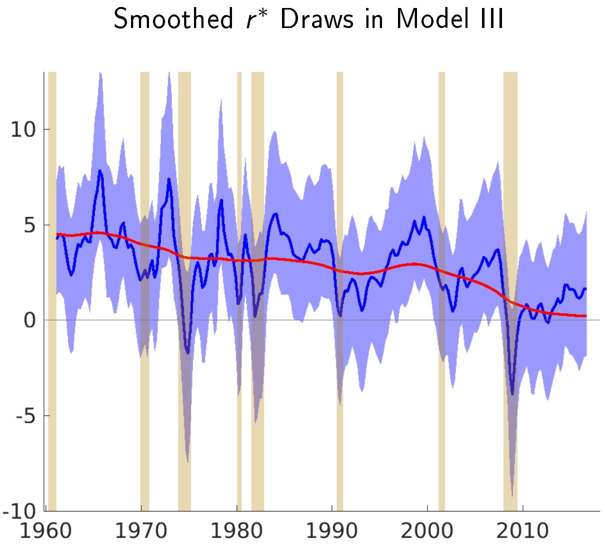


Model II: ρ_g and μ_g estimated, $\rho_z = 1$

Figure: ρ_g

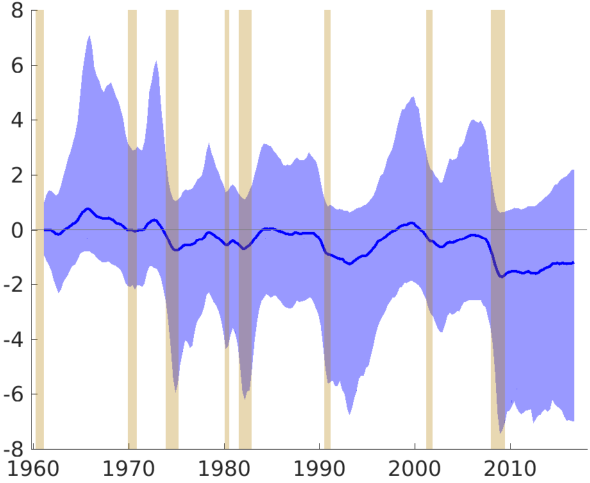


Model III: $\rho_g = 1$, ρ_z estimated

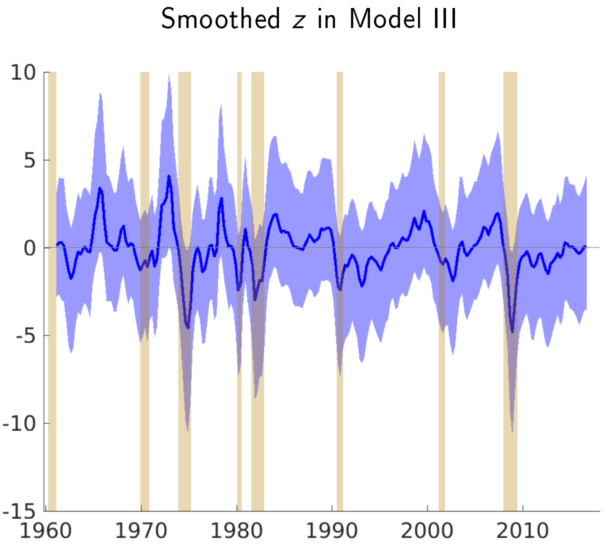


Model I: $\rho_g = 1$, $\rho_z = 1$

Smoothed z in Model I



Model III: $\rho_g = 1$, ρ_z estimated



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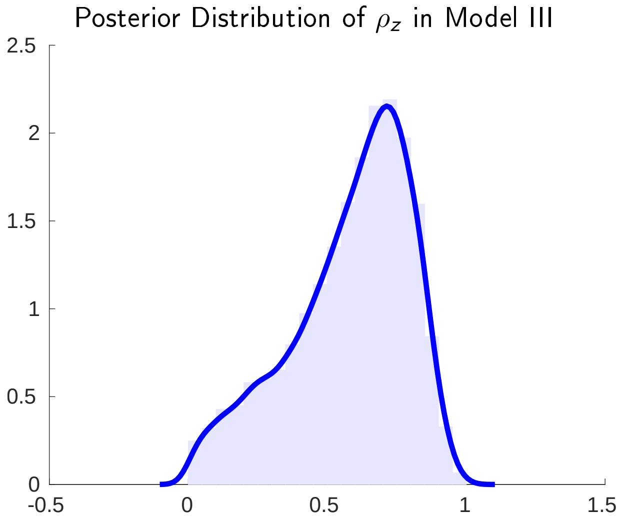


Model I vs. Model III

- The difference between the median path of r^* in Model I and Model II doesn't seem large.
- Model III looks different than Model I (in terms of the median path).
- The only difference between the models is that Model I has a degenerate prior on $\rho_z \equiv 1$.

Model Comparison

- Bayes Factor for nested models reduces to the Savage-Dickey density ratio (Dickey, 1971).

$$B_{III,I} = \frac{pr(Y|M_{III})}{pr(Y|M_I)} = \frac{p_{III}(\rho_z = 1)}{p_{III}(\rho_z = 1|Y)}$$

Savage-Dickey Density Ratio

Figure: Prior and Post. ρ_z

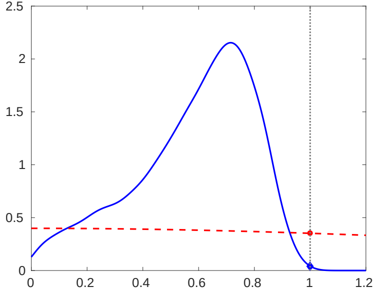
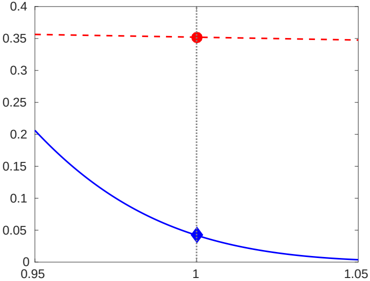


Figure: Area around $\rho_z = 1$



$$B_{III,I} = \frac{p_{III}(\rho_z = 1)}{p_{III}(\rho_z = 1|Y)} = 8.4$$

Model Comparison

	Bayesian		MLE	
	Log Marg. Like.	BF	Log. Like.	BIC
Baseline	-533	0.1	-518	1088
Alternative	-526	10.2	-517	1093

Connection to Theory

- The r^* equation is a linearized euler equation, we can denote the stochastic discount factor (SDF) by S:

$$e^{-r_t^*} = E_t\left[S_{t+1}\right]$$

- Consider an SDF that differs from that of log-utility by an extra term $(\widetilde{Z})$ as in Campbell and Cochrane, Epstein-Zin, etc. Then we have

$$r_t^* = \log E_t\left[\frac{C_{t+1}}{C_t}\widetilde{Z}_{t+1}\right] = \log E_t\left[e^{g_{t+1}+\widetilde{z}_{t+1}}\right] \approx E_t\left[g_{t+1} + z_{t+1}\right]$$

- z_t can be interpreted as an asset pricing term that measures the separation from log-utility of the SDF. We can give z this "headwinds" interpretation.

Headwinds

- Frequently, “headwinds” are cited as a reason for why the level of r^* is still so low.
  - “...lingering sense of caution on the part of households and businesses in the wake of the trauma of the Great Recession.” (Yellen, 3/3/17)
- z_t is the “special sauce” (Williams, 2015 Brookings), it is all the things that are not economic growth.
  - There is nothing that says the components have to be stationary or persistent.
  - In the current version of the model, there is no data specifically aimed at estimating z_t.
  - z_t soaks up the variation in the rate gap that doesn't appear to be linked to growth.
  - Headwinds seem like they should be temporary

Conclusion

- Single-step Bayesian estimation with less informative priors shows deeper drops and subsequent recoveries after recessions, in contrast to multi-step MLE results.
- When z is not assumed to be a random walk, we estimate a greater recovery of r^* since the lows of the great recession, reaching closer to 2% at the end of 2016Q3.
- Our conclusion is that permanent shocks to z (and thus, in our minds, the SDF) are needed to produce a persistent low level of r^* after the great recession.
- The dynamics of z are hard to estimate with this data.
  - More structure around z may be helpful.

APPENDIX

Parameter Estimates

	Bayesian		MLE	
	Baseline	Alternative	Baseline	Alternative
a_1	1.251 [0.97,1.51]	1.270 [0.84,1.52]	1.531 [1.36,1.70]	1.530 [1.36,1.70]
a_2	-0.364 [-0.58,-0.07]	-0.348 [-0.59,0.05]	-0.589 [-0.76,-0.42]	-0.587 [-0.76,-0.41]
a_r	-0.113 [-0.19,-0.06]	-0.093 [-0.18,-0.06]	-0.070 [-0.10,-0.04]	-0.067 [-0.10,-0.04]
b_1	0.682 [0.57,0.79]	0.665 [0.56,0.78]	0.671 [0.60,0.74]	0.670 [0.60,0.74]
b_Y	0.051 [0.03,0.13]	0.071 [0.04,0.15]	0.077 [0.04,0.12]	0.079 [0.04,0.12]
σ_1	0.412 [0.11,0.66]	0.279 [0.08,0.57]	0.355 [0.21,0.50]	0.365 [0.21,0.52]
σ_2	0.794 [0.74,0.87]	0.795 [0.74,0.86]	0.791 [0.75,0.83]	0.791 [0.75,0.83]
σ_3	0.149 [0.07,1.69]	1.755 [0.67,3.95]	0.160 [0.10,0.23]	0.172 [0.10,0.25]
σ_4	0.564 [0.1,0.64]	0.580 [0.25,0.65]	0.571 [0.48,0.66]	0.567 [0.47,0.66]
σ_5	0.036 [0.02,0.13]	0.035 [0.02,0.11]	0.030 [0.02,0.03]	0.030 [0.02,0.03]
ρ_z	1*	0.789 [0.31,0.89]	1*	0.916 [0.77,1.06]

Laubach-Williams 3-part Estimation

Step 1 Hold g constant, drop real rate gap from model, then:

- Get estimate of potential output, $\widehat{y}^*$, compute $\Delta\widehat{y}^*$
- λ_g is equal to Andrews and Ploberger (1994) exponential Wald statistic for the test of a structural break at unknown date in $\Delta\widehat{y}^*$.

Step 2 Impose λ_g value from Step 1, include real rate gap, but hold z constant, then:

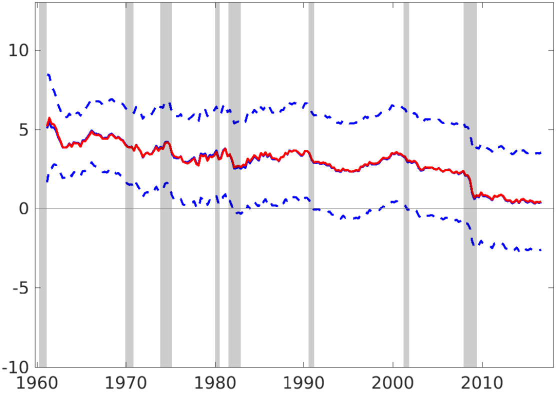
- Estimate the simplified model
- λ_z is equal to Wald statistic for the test of a shift in the intercept of the IS equation.

Step 3 Impose λ_g from Step 1 and λ_z from Step 2, and estimate the remaining parameters by MLE.

Back

HLW replication

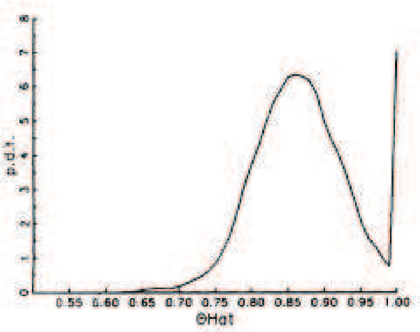
One-sided Bayesian estimate (blue) and HLW results (red)

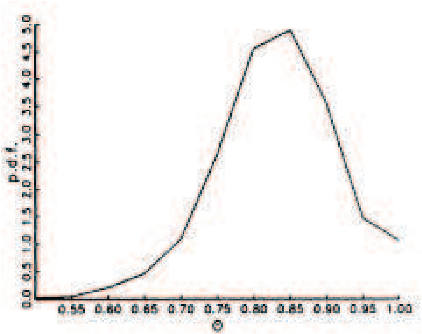


back

DeJong and Whiteman (1993)

Monte Carlo exercise where the true parameter value is 0.85, $T = 100$
Sample distribution of MLE estimate (left) and posterior distribution (right)





back

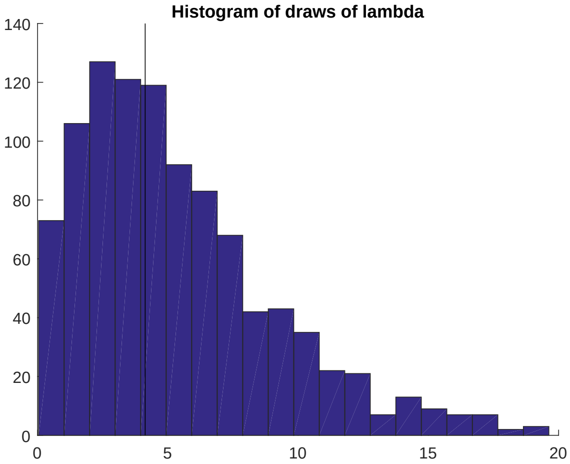
Stock and Watson '98

Our estimation technique does not suffer from the pile-up problem.

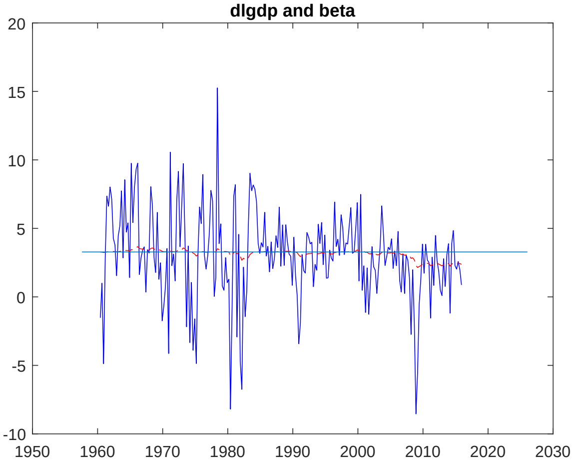
To illustrate this: Consider Stock and Watson 1998 local level model of log GDP growth.

$$
\begin{aligned}
\Delta y_t &= \beta_t + u_t \\
\beta_t &= \beta_{t-1} + \frac{\lambda}{T}\eta_t \\
u_t &= a_1 u_{t-1} + a_2 u_{t-2} + a_3 u_{t-3} + a_4 u_{t-4} + \varepsilon_t
\end{aligned}
$$

Replication Stock and Watson 98



Replication Stock and Watson 98



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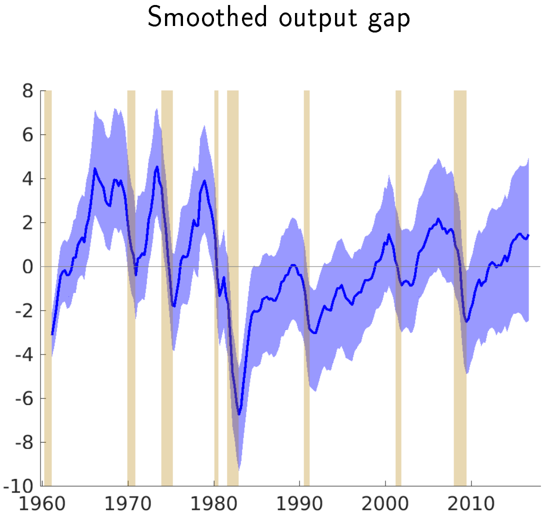
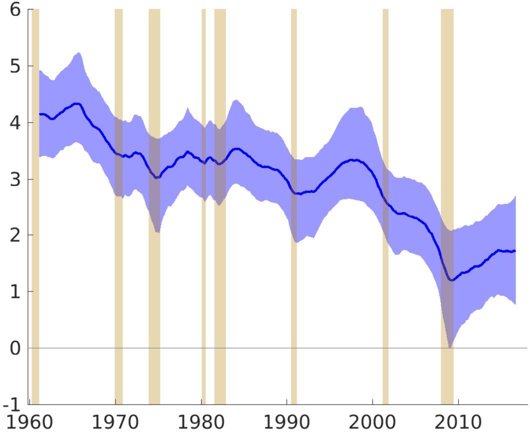
Key Posterior Estimates From Each Model

Model I: $\rho_g = 1$, $\rho_z = 1$



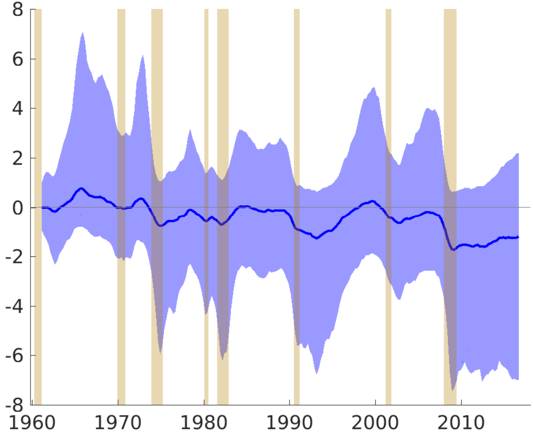
Model I: $\rho_g = 1$, $\rho_z = 1$

Smoothed g path



Model I: $\rho_g = 1$, $\rho_z = 1$

Smoothed z path



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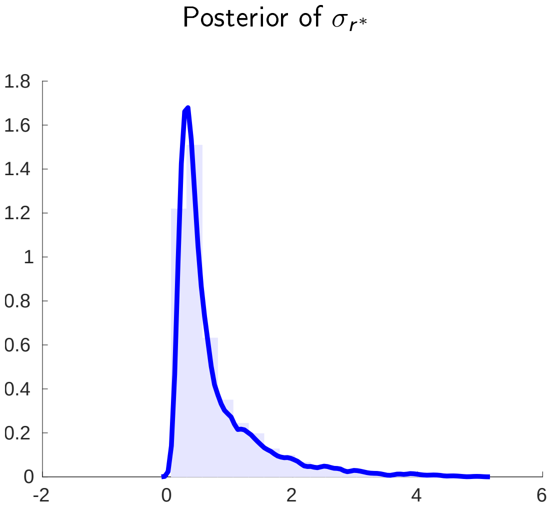
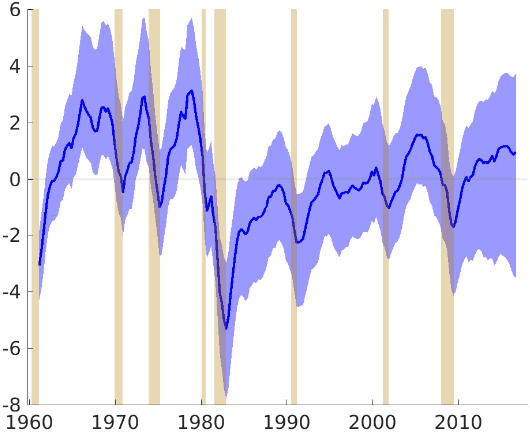


Model II: ρ_g and μ_g estimated, $\rho_z = 1$

Smoothed output gap



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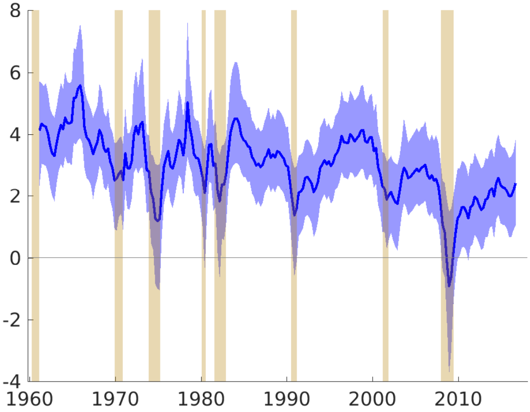
Smoothed g path



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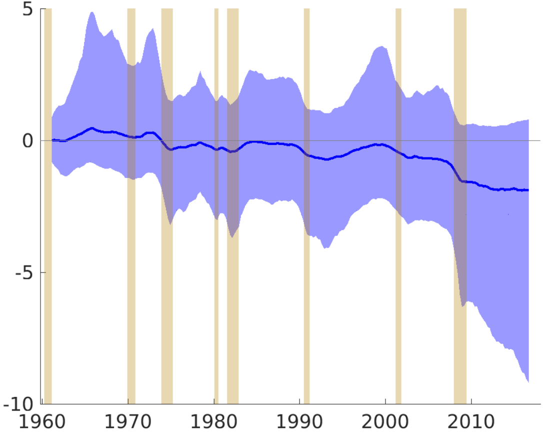

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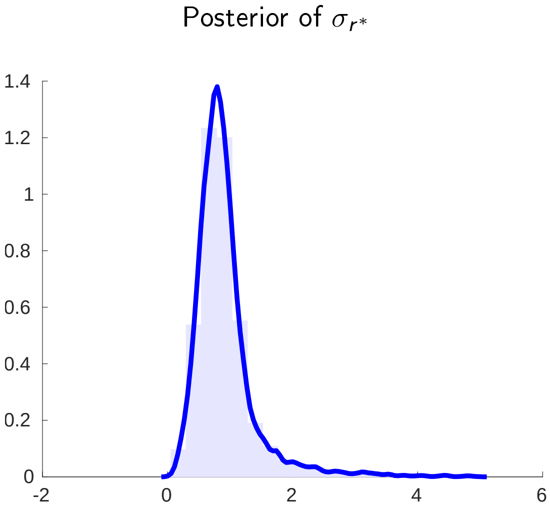

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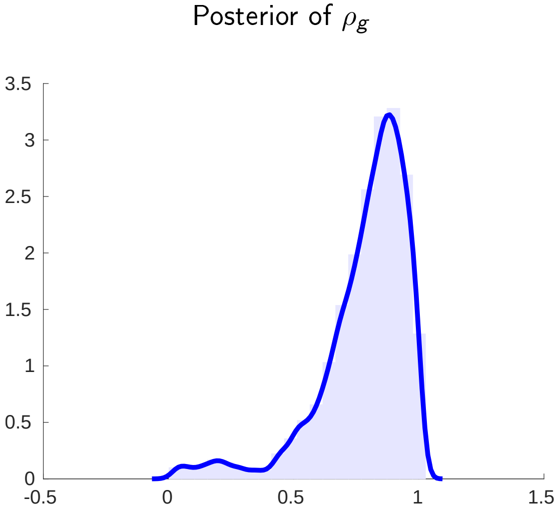
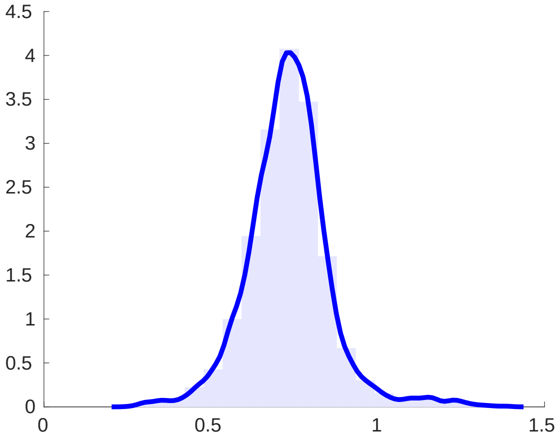


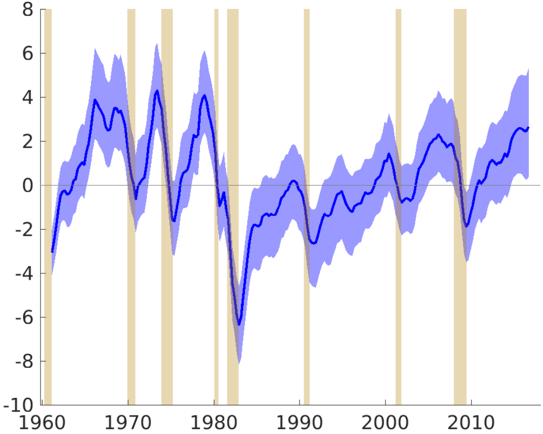
Model II: ρ_g and μ_g estimated, $\rho_z = 1$

Posterior of μ_g (Quarterly)



Model III: $\rho_g = 1$, ρ_z estimated

Smoothed output gap



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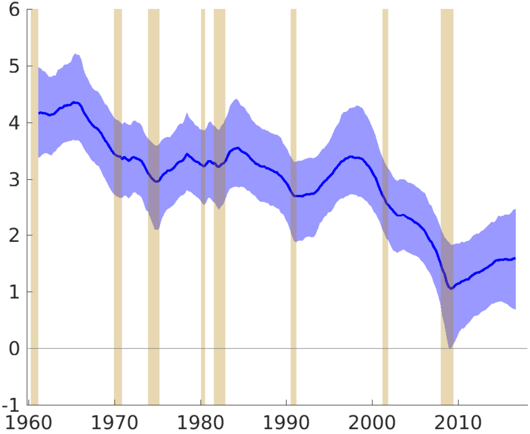
Smoothed g path



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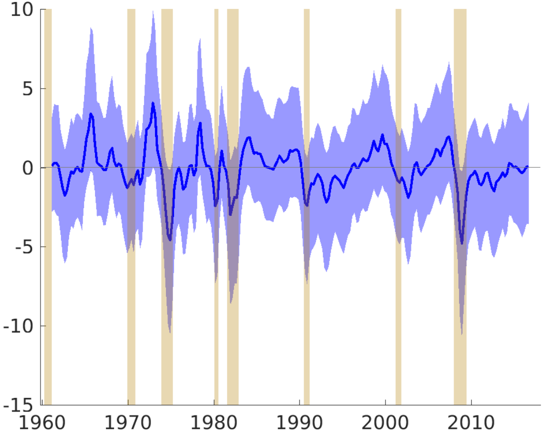

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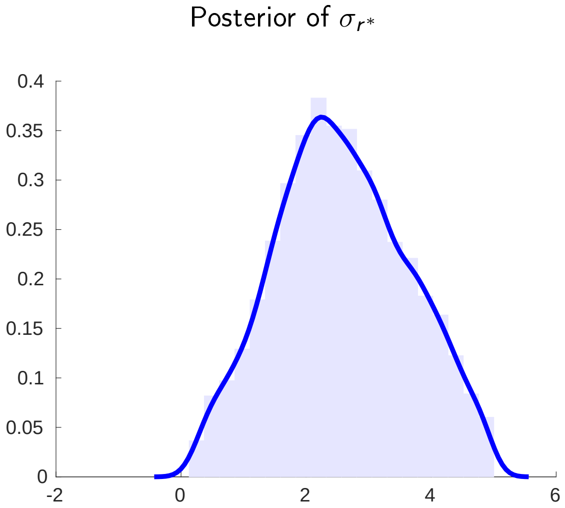

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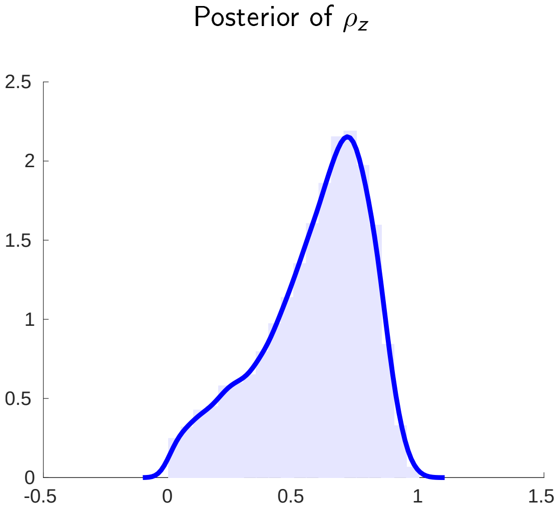
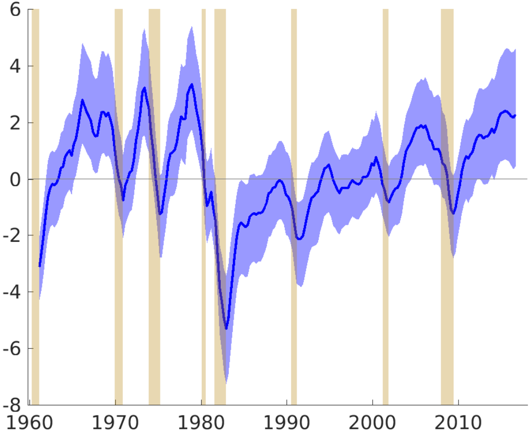


Model IV: ρ_g, μ_g and ρ_z estimated

Smoothed output gap



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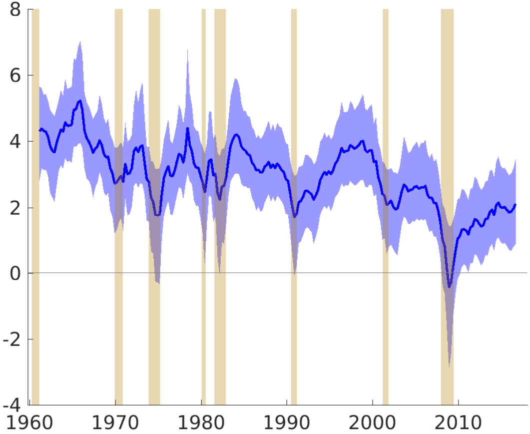
Smoothed g path



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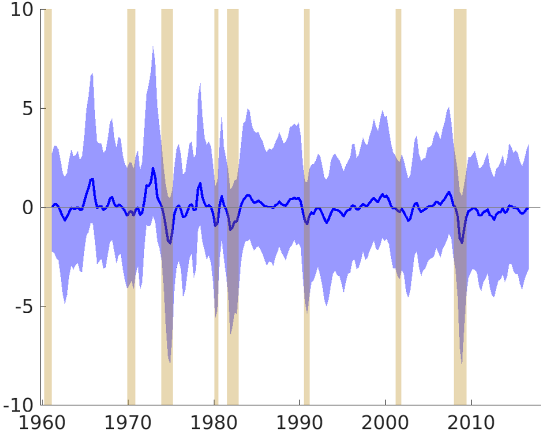

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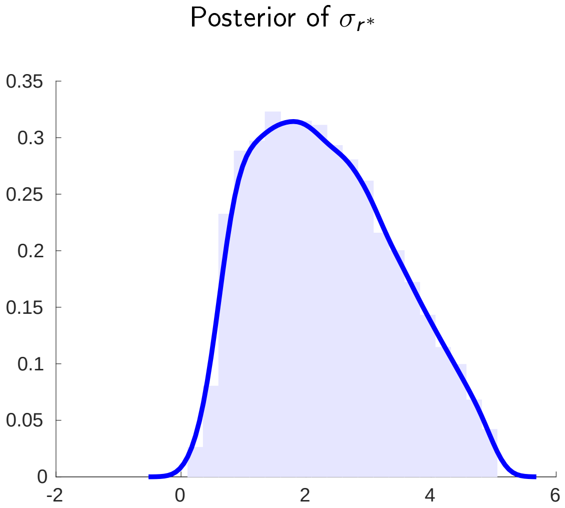

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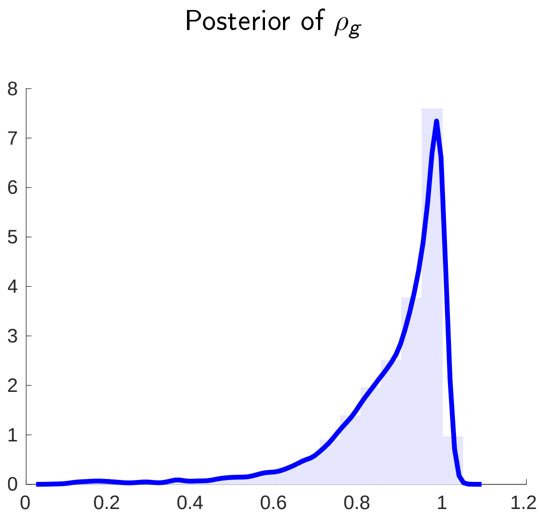

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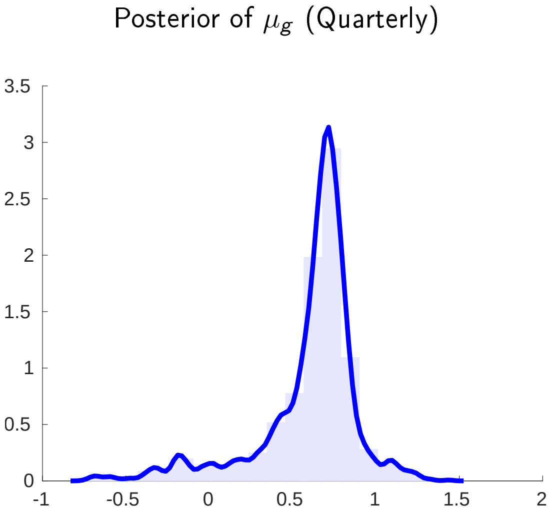
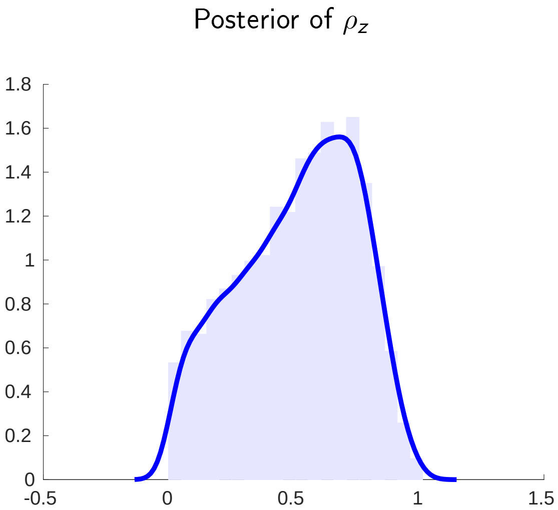
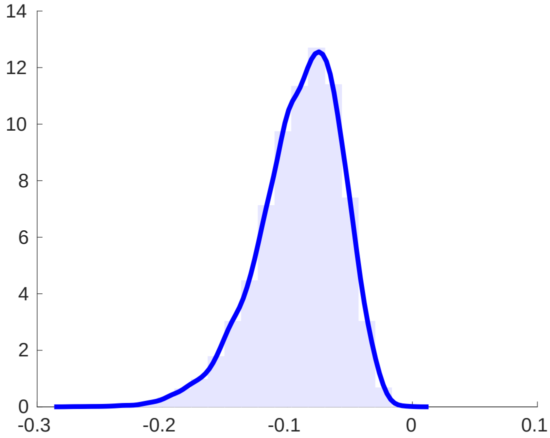


Model IV: ρ_g, μ_g and ρ_z estimated

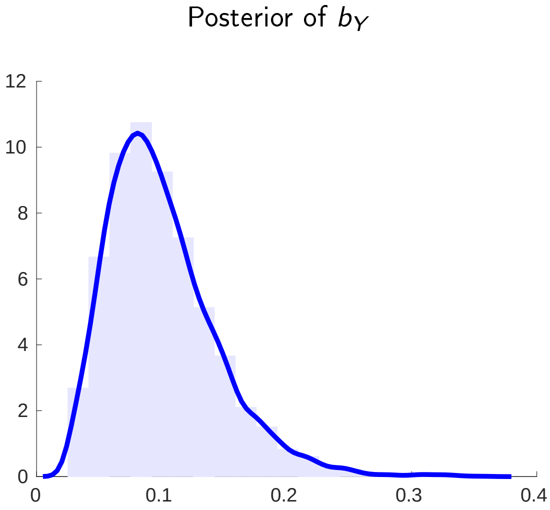


Model IV: ρ_g, μ_g and ρ_z estimated

Posterior of a_r



Model IV: ρ_g, μ_g and ρ_z estimated



Model IV: ρ_g, μ_g and ρ_z estimated

Posterior of b_1